Technical Note Number
62-38

THE RELATIVE IMPORTANCE OF DENSITY, SPECIFIC IMPULSE AND OTHER SOLID PROPELLANT PROPERTIES IN THE FRAME OF LONG-TERM RESEARCH GOALS

William E. Gordon

IDA *Institute for Defense Analyses*

440 200

Research and Engineering Support Division

August 1962

UBG-62-521
Copy 16 of 150

TECHNICAL NOTE 62-38

THE RELATIVE IMPORTANCE OF DENSITY, SPECIFIC IMPULSE AND OTHER SOLID PROPELLANT PROPERTIES IN THE FRAME OF LONG-TERM RESEARCH GOALS

William E. Gordon

August 1962

Institute for Defense Analyses
Research and Engineering Support Division
Contract SD-50

TABLE OF CONTENTS

I. CONCLUSIONS

A. The value of an analysis such as this to determine the relative importance of propellant density and specific impulse is limited because of the interplay of unrelated variables. Quantitative criteria, such as the index n in the expression $I_{sp}\rho^n$, can never be applied with generality. Analysis can only indicate when density is a major concern and when its effect is minor.

B. High density is a desirable property of a solid propellant in all applications.

C. The penalty suffered by a propellant because of low density ranges from being completely negligible to very serious. The magnitude of the penalty depends on:

1. the particular mission and design of the stage of the rocket in which the propellant is to be used,
2. the other properties of the propellant besides density.

D. A low-density propellant is at its greatest disadvantage in a sea-level, first-stage oooster, regardless of the types of constraints (e.g., constant-weight or constant-volume). A solid propellant with a density as low as 1.0 g/cm^3 is virtually ruled out for this application, at least at present.

E. The constant-volume-type constraint exacts a greater penalty on a propellant because of low density than does a constant-weight-type constraint. In terms of equivalent specific impulse, the penalty for a given drop in density will usually be from two to five times as great in the constant-volume case as in the constant-weight case.

F. On the very tenable premise that new high specific-impulse solid propellants will be used only in upper stages in the foreseeable future, only constant-weight constraint needs to be considered in respect to their use.

G. The penalty for low-propellant density in an upper stage (with constant-weight constraint) is in direct proportion to the combustion chamber pressure. With present structural materials, and with a pressure of 200 psi, a propellant of density 1.0 g/cm^3 loses about 5 sec of equivalent specific impulse compared to a propellant of density 1.6 g/cm^3. At 1000 psi chamber pressure, the penalty is five times as great, or 25 sec. It is imperative, therefore, that low-density propellants have good low-pressure burning properties; but if they have, the effect of density can, even now, be slight.

H. Low combustion temperature and low erosion and corrosion tendencies of the exhaust gases -- properties that are often concommitant -- are favorable to a low nozzle weight. Good properties in these respects can more than offset the effect of a considerable drop in density.

I. For space applications, a low absolute burning rate is also desirable. This can serve to mitigate the effect of low density.

J. Future improvements in structural materials will constantly reduce the importance of propellant density for upper-stage use. Eventually, we can expect density to be an insignificant factor in this application.

K. Structure weight reduction, itself, will continually diminish in importance as a research goal. This leaves high specific impulse *per se* as the outstanding objective of long-term, solid-propellant research.

II. INTRODUCTION

A. Nature of the Problem

The reason the relative importance of propellant density and specific impulse has sometimes been in dispute is that it is extremely sensitive to the context -- either hypothetical or real -- in which the question is examined. To fix the basis of comparison, interest is usually centered on a given rocket stage (i) with a specific mission to perform, e.g., to impart a total velocity increment ΔV_i to a payload $M_{u,i}$. The following question is then asked: "If the size of the stage and the mission requirements are fixed, how many seconds of specific impulse (- or +) must be traded for a given change in propellant density (+ or -)?" The answer to this question, it turns out, depends very strongly on what is meant by "size" -- whether it is weight or volume -- and on what the mission requirements are.

The basic equation is

$$\Delta V_i = g\, I_{sp,i}\, \ln R_i \qquad (1)$$

where g is the acceleration of gravity (constant), $I_{sp,i}$ is the specific impulse and R_i is the mass ratio, defined as total weight when the i^{th} stage is ignited (and after separation of the prior stage) divided by the weight when it has burned out (and before separation of the subsequent stage).

The way in which various factors enter Eq. (1) can be seen by writing R_i in an expanded form, as follows:

$$\Delta V_i = g\, I_{sp,i}\, ln \frac{1}{1 - \Lambda_i(1 - \zeta_i)} \qquad (2)$$

Here, Λ_i is the propellant weight fraction in the i^{th} stage and ζ_i is the "payload ratio," the ratio of weight of the payload (everything above the i^{th} stage) to the total weight of the rocket down (from the top) to, and including the i^{th} stage. If we let $M_{u,i}$ represent the payload weight, $M_{m,i}$ the total stage weight, $M_{p,i}$ the propellant weight, and $M_{s,i}$ the structure weight, the symbols Λ_i and ζ_i in Eq. (2) can be written

$$\Lambda_i = \frac{M_{p,i}}{M_{p,i} + M_{s,i}}$$

$$\zeta_i = \frac{M_{u,i}}{M_{u,i} + M_{m,i}}$$

where $M_{m,i} = M_{p,i} + M_{s,i}$

Equation (2) can then be expanded, as follows, in order to further reveal the significance of several variables:

$$\Delta V_i = g\, I_{sp,i}\, \ln \frac{1}{1 - \Lambda_i \cdot \frac{M_{m,i}}{M_{m,i} + M_{u,i}}} \qquad (3)$$

Here, it will be noted, the magnitudes ΔV_i and $M_{u,i}$ serve -- so far as can be done by mathematics -- to define the mission. The term $M_{m,i}/(M_{m,i} + M_{u,i})$ in the denominator is an expression for the weight of the stage, expressed in terms of payload weight. The factors $I_{sp,i}$ and Λ_i may be regarded as primarily determined by the properties of the propellant and the structural techniques. We may then consider that the rocket designer starts with the mission requirements (ΔV_i and $M_{u,i}$), puts in values for $I_{sp,i}$ and Λ_i appropriate to the propellant he is going to use and the current structural state-of-the-art, and solves the equation for $M_{m,i}$, the total stage weight required to accomplish the mission.

If $I_{sp,i}$ and Λ_i were single-valued propellant properties, the rocket designer's job would be much easier than it is in practice. The specific impulse, indeed, is not very sensitive to various mission requirements and structure considerations; and, for a first approximation, it can be assigned a nominal value that derives only from intrinsic properties of the propellant. On the other hand, the propellant mass fraction Λ_i, although it depends partly on the intrinsic density of the propellant, is strongly influenced by other magnitudes and by the nature of the mission itself.

In the first place, as is well known, the propellant fraction depends somewhat on the size of the rocket motor: it generally tends to be greater, the larger the motor, other factors being equal. It also makes a great deal of difference to the value of Λ whether the stage burns near the ground, at a high ambient pressure, or in space, where the exhaust gases emerge in a vacuum. When the exhaust is near sea-level pressure, the chamber pressure must be high in order to get the necessary expansion. This means a strong, heavy case. In the vacuum of space, on the other hand, the expansion ratio is determined solely by the nozzle geometry; and the chamber pressure -- being set by other factors -- is generally lower. In consequence, a space engine can be more lightly constructed than a sea-level booster. The rocket thrust also determines the size of nozzle needed to accommodate the flow of propellant gases. A rocket rising vertically near the earth should have high acceleration to minimize the effect of gravity; the nozzle, then, must be large, and Λ tends to be low on this account. A rocket unaffected by gravity, on the other hand, can operate with low thrust -- and, consequently, with a small nozzle if this meets other requirements.

To further complicate matters, Λ depends on the burning properties of the propellant. If the exhaust gases are highly erosive or corrosive, a heavy tungsten throat insert may be required in place of graphite. The flow rate of exhaust gases is also determined to a greater or lesser extent -- depending on what other factors are involved -- by the

absolute burning rate of the propellant. A faster burning propellant may require a larger nozzle; although this is not always the case.

Finally, A depends on the materials of construction, the shape of the pressure vessel, and the structural features needed to meet mechanical requirements such as stiffness. These factors may or may not interact with the propellant properties.

Propellant density is therefore deeply buried in a welter of other variables. And only by freezing the action of all the other factors can the effect of density be extracted. This must always be considered an arbitrary and unrealistic procedure. For, when one propellant is substituted for another in practice, several factors besides density usually change at the same time. Some of these may have an even greater effect on A than density; and their net influence may be either of the same sign or of opposite sign. A complete redesign of the rocket motor is needed when such a change is made -- usually, in fact, not simply redesign, but redevelopment.

What, therefore, is to be gained by analysis of this problem? Obviously, only some heavily qualified and rather vague generalities. But the net conclusion, even of this preliminary examination, is worth attention: The importance of density is bound up with the influence of so many other propellant characteristics, as well as with the mission and system requirements, that it would be very unwise to focus attention on it during the early stages of research on new propellants when these other properties, both favorable and unfavorable, are still unknown, and when the end uses can still only be conjectured on.

B. The Constraints

The effect of density is usually derived under either a constant-weight or constant-volume constraint. In the constant-weight case, only the term Λ in Eq. (3) is sensibly affected by a change in density; in the constant-volume case, both Λ and M_m will be subject to change.

Mathematically, there is no difficulty in deriving the results. But to do so, further assumptions -- or constraints -- have to be imposed. Due to the important way in which propellant characteristics besides density affect Λ (as outlined in the foregoing section), these ad hoc assumptions may be just as important in their own right as the main constraint of fixed volume or fixed weight.

One implicit constraint is chamber pressure. Since the only way in which propellant density enters the propulsion equation is through its effect on the proportional distribution of weight as between propellant and structural hardware, some assumption must be made about the chamber pressure because pressure is the major criterion for the strength of the motor casing required. For want of a more general or more plausible assumption, pressure is usually considered to remain unchanged when the propellant density is altered.

Another implicit assumption is that the total weight of all the inert parts other than the motor case remains constant. Since the weight of these other parts often totals to two or three times that of the motor case (in upper stages), and since these components are strongly affected by the propellant characteristics (other than density),

fixed inerts weight can almost never be maintained in practice when one propellant is substituted for another. The constant-inerts-weight constraint is therefore highly artificial; but -- as in the case of chamber pressure -- there seems to be no better alternative, since the propellant characteristics that influence these weight factors are not functionally related to density in any general way.

For the sake of brevity, and to be consistent with conventional practice, we shall further perpetrate the use of the terms "weight-limited" and "volume-limited" to designate the two major constraint groups. But it must be borne in mind that these are merely labels. The risk taken in using these terms is that they might be construed as if to cover important practical situations in rocket design. Unfortunately, situations as simple as this seldom arise in the world of reality.

C. The Density-Exponent Expression

For purposes of a performance criterion or a tradeoff relation, specific impulse I_{sp} and density ρ are often combined in an expression

$$I_{sp}\rho^n$$

The meaning here is that if two propellants of different I_{sp} and ρ are considered, their relative performance can be gauged by the value of $I_{sp}\rho^n$. Or, from another standpoint, if the value of this quantity is equal for two propellants, they will give equal performance under conditions for which the expression is valid.

It is this last phrase "under which the expression is valid" that points up the weakness in this simple form of criterion. The qualifications restricting the range of validity for any particular value of the exponent are so confining as to make it almost useless as a quantitative indicator. The exponent n may have values ranging from almost 0 to almost 1.0, depending on the constraints imposed when the expression for n is derived, and especially on the figures that are substituted in this expression for the various rocket parameters in order to assign to n a numerical value.

Even within the narrow range of such restrictions, the expression $I_{sp}\rho^n$ cannot be used to compare propellants of widely differing density. This is because n is derived on the basis of differential changes in I_{sp} and ρ as follows:

$$n = - \frac{\left(\frac{\partial \Delta V}{\partial \ln \rho}\right)_{I_{sp}}}{\left(\frac{\partial \Delta V}{\partial \ln I}\right)_{\rho}} \tag{4}$$

where ΔV is the velocity increase of the rocket due to the burning of the propellant in the particular stage in question. (Equation (4) is derived from the significance ascribed to $I_{sp}\rho^n$ as a figure of merit. It may be regarded as a definition of n consistent with such an ascription.) In general, the expression for n as defined in Eq. (4) involves parameters that themselves depend on ρ; hence only an instantaneous value of n at a particular density can be obtained.

Equation (4), it may be incidentally noted, demonstrates the "tradeoff" significance of $\underline{n}$. Here $\underline{n}$ appears as the ratio of the change in velocity for a given percentage increment in density to the change in velocity for an equal percentage increment in impulse. In this sense, $\underline{n}$ may be regarded as the ratio of a density "index" to an impulse index. However appealing it is to have a single number representing the relative importance of impulse and density, the practical use of the concept is severely limited. In fact, generalizations on the basis of a particular value for $\underline{n}$ are always misleading and can result in seriously erroneous conclusions.

The important information to be gained from the $I_{sp}\rho^n$ concept is not a precise knowledge of the significance of propellant density, but rather a general idea of when density plays a prominent role and when it plays a minor one. Table I can be used to gauge the significance of various values for $\underline{n}$ derived in the following sections. The values ΔI_{sp} given in the table are the tradeoff increment in specific impulse (+ or -) corresponding to a change of 0.1 g/cm^3 in density (- or +). The calculations are based on a nominal reference propellant with I_{sp} = 260 sec and ρ = 1.60 g/cm^3. It will be noted that this table cannot be used to extrapolate for large density changes greater than, say, 0.3 or 0.4 g/cm^3.

Table I

APPROXIMATE TRADEOFF IN SPECIFIC IMPULSE FOR A DENSITY CHANGE OF 0.10 g/cm^3

n	ΔI_{sp} (sec)
0.05	0.8
0.10	1.6
0.25	4.0
0.50	8.0
0.75	12.0
1.00	16.0

It is the purpose in what follows, first, to examine critically the generalizations that have often been made on the basis of derived expressions for n. Secondly, we shall study in detail, from a strictly quantitative viewpoint, the effect of gross density changes under a constant-weight comparison criterion. Finally, we shall attempt to draw some conclusions from this, and find a basis on which the promise or lack of promise of new developments in low-density solid propellants may be judged.

III. DIFFERENTIAL EXPONENT n FOR A WEIGHT-LIMITED SYSTEM

Upon differentiation of Eq. (3) in order to find the expression for n according to Eq. (4), the result for a constant-weight constraint -- plus the implicit constant-pressure and constant-inerts-weight constraints, as previously noted -- may be shown to be

$$n = \frac{\phi}{\phi + 1} \cdot \frac{(R - 1)}{\ln R}$$

$$\text{where } \phi = \frac{\text{weight of volume-dependent structure (case)}}{\text{weight of the propellant}} \tag{5}$$

In this equation, n is modulated by ϕ, the volume-dependent structure factor. When ϕ is zero, n is also zero. The factor ϕ depends inversely on the strength-to-density ratio of the case material; hence as lighter and stronger materials are found, the effect of density will be reduced.

The most important effect on the value of ϕ (and hence on n), however, comes from the chamber pressure P_c, since ϕ depends on P_c directly. Present technology, with acceptable safety factors, gives approximately

$$\phi = 10^{-4} P_c (\text{lb/in}^2) \tag{6}$$

Thus, for 1000 psi chamber pressure ϕ has a value of about 0.1, whereas for 100 psi the value drops to 0.01. This means that density will be more important in sea-level boosters where the chamber pressure is high than in upper stages where the pressure is generally lower. However, it is to be

noted that a low-density propellant must then have satisfactory burning properties at low pressures if it is to be useful in upper-stage applications.

Aside from its dependence on chamber pressure, the exponent $\underline{n}$ is also seen to depend on the mass ratio R of the rocket stage in question. This again illustrates why it is impossible to assign to $\underline{n}$ any fixed numerical value.

Figure 1 shows a plot of $\underline{n}$ vs R for various values of ϕ. Since in practice R ranges from about 2 to 5 and ϕ takes on values from 0.01 to 0.1 (depending on pressure), $\underline{n}$, for the weight-limited case, will range from truly negligible values up to about 0.25.

When $\underline{n}$ is as high as 0.25, the effect of density becomes fairly sizeable. In Table I, the specific impulse tradeoff for a density change of 0.1 g/cm^3 is 4 sec when $n = 0.25$. Thus, for large changes in density, the effect could be quite serious.

The value $n = 0.25$, however, is extreme for this case. If an "average" value can be considered to have meaning, one would be justified to take, say, $n = 0.10$ to 0.15. This would mean a tradeoff of about 2 sec in specific impulse for a density change of 0.1 g/cm^3.

IV. DIFFERENTIAL EXPONENT n FOR A VOLUME-LIMITED SYSTEM

It is to be expected that density will be more important in a volume-limited than in a weight-limited system. Here again, however, a low-density propellant suffers a smaller disadvantage in upper stages than in lower stages. But the reason is different. For, while the performance of the particular stage in question is reduced to the same degree by substitution of a lower density propellant no matter where it is located in the rocket, the decrease in velocity is partly compensated when the low-density propellant is used in an upper stage by the increase in the velocity increments supplied by stages below this because of the lightened burden they have to accelerate. This is indeed an interesting result in principle, but probably not so important in practice because volume limitation -- such as in the POLARIS missile -- usually applies more severely to the booster than it does to the upper stages of a rocket.

For a volume-limited, first-stage booster, the expression for n is found to involve* only the mass ratio R as follows:

$$n = \frac{R - 1}{R \ln R} \tag{7}$$

For an upper stage (which we shall designate the i^{th} stage), n is less than the value given by Eq. (7) because of the reduced-payload effect on the stages below i as explained above. In this upper-stage case, n is given by the following expression:

$$n_i = \frac{R_i - 1}{R_i \ln R_i} \left[1 - \sum_{j=1}^{(i-1)} \frac{I_{sp,j}}{I_{sp,i}} (R_j - 1) \zeta_j \zeta_{j+1} \cdots \zeta_{i-1} \right] \quad (8)$$

where ζ_j etc. are the so-called "payload ratios" for the stages below the i^{th} one where the propellant change is made. The payload ratio ζ is defined as the ratio of the total of the weights of all stages (M_p) above the given stage (including that of the ultimate payload M_N) to the total of the weights of all stages above together with that of the given stage, i.e.,

$$\zeta_j = \sum_{q=j+1}^{N} M_q \Big/ \sum_{p=j}^{N} M_p \quad (9)$$

Values for n are plotted as a function of R in Figure 1. To evaluate n for an upper stage (i) from Eq. (8), specific impulse in the lower stages was assumed to be equal to that in the stage in question (i.e., $I_{sp,j} = I_{sp,i}$), the mass ratio for each of these stages was assumed to be equal to 3.0, and the payload ratio was assumed to be the same in all the lower stages and to have a value 0.25.

From Figure 1, it is seen that, within the range of R that is common for most practical rockets (i.e., R = 2 to 5), the differential density exponent n for the booster lies between 0.7 and 0.5. For a second-stage motor, the figure is equal to about one-half the value for a booster stage,

i.e., n ranges from 0.35 to 0.25. For a third-stage motor, it is still lower, lying between 0.26 and 0.19. For stages above the third, the n-values differ little from those for the third stage. (See the curve for the fourth stage in Figure 1.) For volume-limited systems, therefore, density has a pronounced influence on performance in a booster (first) stage; but it is not so important in the second and higher stages. However, as reference to Table I will illustrate, the effect of propellant density in the volume-limited case is never by any means negligible.

An examination of the n-curves for constant-weight and those for constant-volume in Figure 1 may create the impression -- since the two sets of curves overlap -- that the difference between the two constraint conditions becomes small for the upper stages. Actually, this is probably never true because the value of Ø is usually smaller, the higher the stage. This is because chamber pressure is generally lower in the higher stages, and Ø bears a direct relation to pressure. In most actual cases, it is probable that n would be anywhere from two to five times larger for the constant-volume case than for the constant-weight case.

In view of the range in values of n seen in Figure 1, it is difficult to see a reason for choosing $n = 1$ to represent any actual situation in rocketry. And yet often when the density factor is considered in making comparisons of propellants, it is introduced through the quantity known as "density-impulse" or "impulse-density" $I_{sp}\rho$,

i.e., the exponent n is assumed equal to one. From the foregoing analysis it is clear that $n = 1$ is an upper limit; this is the case for constant-volume constraint when $R = 1$, i.e., when the propellant represents a negligible fraction of the total weight.

The quantity $I_{sp}\rho$ is proportional to the total impulse available from a fixed volume of propellant. When the propellant weight-fraction approaches zero, the velocity increment ΔV is just proportional to the total impulse, and, therefore, to $I_{sp}\rho$; but when the propellant comprises an appreciable fraction of the total weight, ΔV is proportional to $I_{sp}\rho^n$ where n is less than one. The reason is as follows: When the density of the propellant changes, the weight of the (constant-volume) rocket also changes by an amount that depends on the proportion of propellant in the total weight (this proportion being reflected in the magnitude of the mass ratio R). Therefore, when density changes either up or down the propellant has either more or less mass (respectively) to accelerate. As a result, the effect on ΔV of a propellant density change is actually always less than it is for the limiting case when the weight of the propellant is negligible compared to the total. This is indeed the rationalization for the descending trend of the curves representing constant-volume constraint in Figure 1.

The "density-impulse" expression $I_{sp}\rho$, therefore, considerably exaggerates the importance of density in all real situations. It is hard to justify any value of n as being "representative" or "average"; but, certainly, the value should never be greater than about 0.7. In

fact, insofar as a quantity of the type $I_{sp}\rho^{n}$ is used to compare advanced high specific-impulse propellants, very much lower values of n, lying between, say, 0.1 and 0.2 would have more validity. The reasons for this will be made clear in the following section.

V. EFFECT OF GROSS DENSITY CHANGES OF THE SOLID PROPELLANT ON PERFORMANCE OF A WEIGHT-LIMITED ROCKET ENGINE

The foregoing sections dealing with the exponent $\underline{n}$ in $I_{sp}\rho^n$ show that a solid propellant of very low density, say 1.0 g/cm^3, will probably never be of interest for booster stages, either weight-limited or volume-limited. This is not so great a handicap as it might seem. The major rocket systems of the future are likely to be multi-stage, each stage making approximately the same contribution to the final payload velocity. Since advanced high-performance propellants are likely to be expensive or in short supply -- probably both -- for some time yet to come, economy will dictate they be used only where the greatest payoff per pound is to be obtained; and this, of course, is in upper stages.* Therefore, at least

---

* As a specific example, consider a three-stage rocket of which the third stage is the ultimate useful payload; that is to say, there are two rocket motors comprising stages No. 1 and No. 2, and No. 3 stage is inert. For simplicity, assume the mass ratio R and the propellant loading fraction Λ are the same for stages No. 1 and No. 2 and equal respectively to 2.72 and 0.90. Also, assume that the same solid propellant is used in both these stages. If a new propellant becomes available that has a 10 per cent higher specific impulse, and other properties essentially the same as the old propellant, it could be added to the first stage, second stage, or both. We wish to calculate for each case the increased weight of payload that could be delivered with the same final velocity, under the restriction that the total weight of the rocket be unchanged.

Analysis shows that the increase in payload under these conditions is the same when either the first-stage motor or the second-stage motor is filled with the new propellant. The efficiency, pounds of added payload per pound of propellant, is therefore in inverse proportion to the weight of propellant in the motor. In this example, the first stage contains 3.33 times as much propellant as the second, and the efficiency is therefore less than one-third as much when the propellant is put in the first stage as in the second. The actual figures are

in the first applications, the new high-I_{sp} propellants will be used only in upper stages regardless of density. And since n is smaller for upper stages, slight density differences between various advanced propellants will be of little consequence.

The remaining question is to determine the effect of gross differences. For this purpose, a new approach to the problem is necessary because the $I_{sp}\rho^n$ concept applies, in principle, only to infinitesimal changes.

Where upper stages are concerned, weight is of much greater significance than volume. For example, so far as the performance of lower stages is affected, the weight of the penultimate stage is indistinguishable from the weight of the ultimate useful-payload stage. The weight at this point is therefore a prime consideration. The volume, on the other hand, is usually small for upper stages anyway, compared, that is, to the volume of the booster. Hence, space

---

0.036 lb added payload per pound of new propellant when the propellant is put in the first stage, and 0.12 lb payload per pound propellant when it is in the second stage. When both motors are filled with new propellant, the payoff is 0.050 lb added payload per pound of new propellant.

In the case of a similar four-stage rocket, the efficiency drops by another factor of 1/3.33 when the new propellant is put in the first stage, that is to 0.011 lb added payload per pound of the new propellant. The efficiency in the first stage here, therefore, is only about 9 per cent of what it is when the propellant is put in the third, or penultimate stage of the same rocket.

(These calculations do not, of course, take account of the effects of gravity or drag. The comparisons would be essentially the same, however, if these refinements were added.)

limitations will restrict the size of the booster but will not usually appreciably affect upper stages. It will, therefore, be necessary to treat only the weight-limited case when we are concerned primarily with the use of new advanced propellants in upper-stage motors.

Consider the problem that a "reference" solid-propellant engine is to be replaced by one containing a propellant of substantially different density. To compare the propellant performance, we shall impose the restriction that the total weight of the rocket be held constant. We shall then calculate the change in specific impulse ΔI_{sp} (tradeoff) necessary to compensate for the change in density, i.e., to keep the velocity change of the rocket ΔV_i due to the burning of the propellant in this engine the same.

The basic formula of rocket engine performance (in the absence of gravity and drag) is

$$\Delta V_i = g\, I_i \ln R_i \tag{10}$$

where we now use I instead of I_{sp} to stand for specific impulse. In Eq. (10) ΔV_i, as before, is the increase in velocity due to burning of propellant in the i^{th} stage rocket motor, g is the acceleration of gravity, and R is defined by

$$R = \frac{\text{total weight at ignition}}{\text{weight at burnout}} = \frac{M_p + M_s + M_u}{M_s + M_u} \tag{11}$$

where M_p, M_s and M_u represent, respectively, the weights of the propellant, the structure and the "payload" for the particular stage in question.

Under the constraint conditions we have set up, the total weight of the rocket is constant, i.e.*,

$$M_p + M_s + M_u = M \text{ (const)}. \tag{12}$$

The change in the velocity increment ΔV_i due to the change in propellant density therefore results solely from the change in the structure weight M_s in the denominator of Eq. (11).

It is convenient to divide M_s into two parts: a part M_x which can be considered as fixed, and a part M_c which varies with propellant density. The quantity M_x includes the weight of the nozzle, the inter-stage structure, and all those parts not primarily affected by the density of the propellant *per se*; and the quantity M_c consists solely of the weight of the case or pressure tank. Symbolically,

$$M_s = M_c(\rho_p) + M_x\text{(const)} \tag{13}$$

where (ρ_p) indicates the dependence of M_c on ρ_p, the propellant density.

In view of Eqs. 12 and 13, Eq. 11 can be written

$$R(\rho_p) = \frac{M\text{(const)}}{M_c(\rho_p) + \left[M_x + M_u\right]\text{(const)}} \tag{14}$$

In Eq. (14), M_c is the only propellant-density-dependent factor on the right, since the numerator and the bracketed quantity in the denominator are considered to be constant under the restrictions of the analysis.

To a good approximation, the weight of the pressure tank is just proportional to propellant volume V_p.

$$\text{i.e.,} \quad M_c \propto V_p \tag{15}$$

However, this relationship (15) can be derived only under the assumption that the chamber pressure P_c is constant, because M_c depends directly on P_c. To include the pressure effect, we should write

$$M_c = S(\text{const}) \cdot P_c \cdot V_p \tag{16}$$

The constant, S, in Eq. (16) is related directly to the density, ρ_s, of the case material and inversely to the strength, σ_s. It also includes a safety factor, F, and a geometrical factor, G. Thus,

$$S = F\, G\, \rho_s/\sigma_s. \tag{17}$$

Since the propellant volume V_p is equal to the propellant weight M_p divided by the density ρ_p, we can write, in view of Eqs. 16 and 17,

$$\phi = \frac{M_c}{M_p} = \frac{S\ P_c}{\rho_p} \tag{18}$$

where, for convenience, we have designated the ratio of the case weight to propellant weight by ϕ.

By rearrangement of Eq. (11), it is possible to show that

$$\frac{R}{R - 1} = \frac{M}{M - M_x - M_u} \cdot (\phi + 1) \tag{19}$$

This equation expresses the functional relationship of R to ϕ. It will be noted that under the constraints we have imposed, M, M_x and M_u are constant. Therefore, in comparing the substituent propellant to the reference propellant under these conditions, we can write

$$\frac{\left(\frac{R}{R - 1}\right)_{\text{substituent}}}{\left(\frac{R}{R - 1}\right)_{\text{reference}}} = \frac{\phi_{sub} + 1}{\phi_{ref} + 1} \tag{20}$$

In view of Eq. (18) and under the assumption of constant chamber pressure P_c,

$$\frac{\phi_{sub}}{\phi_{ref}} = \frac{\rho_{p,ref}}{\rho_{p,sub}} \tag{21}$$

Thus, given values of R and Ø for the reference propellant, we can calculate these quantities for the substituent propellant from Eqs. 20 and 21. Then, from the basic equation (10) we can calculate ΔI_i, the change in specific impulse necessary to compensate for the change in propellant density. This calculation is made by use of the equation

$$\frac{\Delta I_{sub}}{I_{ref}} = \frac{\ln R_{ref} - \ln R_{sub}}{\ln R_{sub}} \qquad (22)$$

The quantity $\Delta I_{sub}(I_{sub} - I_{ref})$ is the specific impulse "tradeoff" for the change in density from that of the reference to that of the substituent propellant.

Figures calculated by the use of Eqs. (18), (20), (21) and (22) are given in Table II for a large range of values of ρ, P_c and R_{ref}, and for two different values of the structural factor S.

The (a) set of figures in Table II is based on an S-value of 1.1×10^{-4} $(g/cm^3)/(lb/in^2)$ and the (b) set of 1.65×10^{-4}. These numbers were arrived at in the following way: The safety factor F was taken as 1.2 (20% safety factor), and the geometrical factor G as 3, which, being the number for a sphere, is the lowest value this factor can have. The value $S = 1.1 \times 10^{-4}$ corresponds to a combination of these values of F and G with a strength-to-weight ratio $\sigma_s/\rho_s = 0.6 \times 10^6$. The 1.1×10^{-4} figure for S may be considered representative of state-of-the-art in high strength/weight case materials and of a

Table II

ADDITIONAL SPECIFIC IMPULSE NEEDED (+ or -) BY A PROPELLANT OF DENSITY ρ TO MATCH THE PERFORMANCE OF A REFERENCE PROPELLANT

Reference Propellant:

I_{sp}(1000 lb/in^2, sea level) = 260 sec

Density (g/cm^3) or Sp Grav = 1.60

Structure Constant S →	(a) $S = 1.1 \times 10^{-4} \left(\frac{g/cm^3}{lb/in^2}\right)$					(b) $S = 1.65 \times 10^{-4} \left(\frac{g/cm^3}{lb/in^2}\right)$				
Mass Ratio R ↓	ρ ↓ \ P_c (lb/in^2) →	100	200	500	1000	ρ \ P_c (lb/in^2)	100	200	500	1000
R_{ref} = 2	0.8	2.6	5.1	12.4	24.0	0.8	3.8	7.6	18.3	34.9
	1.0	1.5	3.0	7.5	14.4	1.0	2.3	4.5	11.0	21.0
	1.2	0.8	1.7	4.2	8.0	1.2	1.3	2.5	6.1	11.7
	1.4	0.4	0.7	1.8	3.4	1.4	0.5	1.1	2.6	5.0
	1.6	0	0	0	0	0	0	0	0	0
	1.8	-0.3	-0.6	-1.4	-2.7	1.8	-0.4	-0.8	-2.0	-3.9
	2.0	-0.5	-1.0	-2.5	-4.8	2.0	-0.8	-1.5	-3.7	-7.0
	2.2	-0.7	-1.4	-3.4	-6.6	2.2	-1.0	-2.1	-5.0	-9.6
R_{ref} = 3	0.8	3.2	6.4	15.6	30.1	0.8	4.8	9.5	23.0	43.6
	1.0	1.9	3.8	9.4	18.2	1.0	2.9	5.7	13.9	26.3
	1.2	1.1	2.1	5.2	10.1	1.2	1.6	3.2	7.7	14.7
	1.4	0.5	0.9	2.2	4.3	1.4	0.7	1.4	3.3	6.3
	1.6	0	0	0	0	1.6	0	0	0	0
	1.8	-0.4	-0.7	-1.8	-3.4	1.8	-0.5	-1.1	-2.6	-4.9
	2.0	-0.6	-1.3	-3.2	-6.1	2.0	-1.0	-1.9	-4.6	-8.9
	2.2	-0.9	-1.8	-4.3	-8.3	2.2	-1.3	-2.6	-6.4	-12.1
R_{ref} = 5	0.8	4.4	8.7	21.1	40.4	0.8	6.6	12.9	31.0	58.1
	1.0	2.6	5.2	12.8	24.5	1.0	4.0	7.8	18.7	35.4
	1.2	1.5	2.9	7.1	13.7	1.2	2.2	4.3	10.5	19.8
	1.4	0.6	1.2	3.1	5.9	1.4	0.9	1.9	4.5	8.6
	1.6	0	0	0	0	1.6	0	0	0	0
	1.8	-0.5	-1.0	-2.4	-4.6	1.8	-0.7	-1.4	-3.5	-6.8
	2.0	-0.9	-1.8	-4.3	-8.4	2.0	-1.3	-2.6	-6.4	-12.2
	2.2	-1.2	-2.4	-5.9	-11.4	2.2	-1.8	-3.6	-8.7	-16.7

lowest permissible safety factor at the present time. The 1.65×10^{-4} value of S represents a more conservative choice, which could allow for a larger safety factor and/or a less favorable vessel shape. Although this second figure might seem unduly conservative, it is thought to be realistic in consideration of the fact that the propellant chamber liner has not been considered as part of the density-sensitive structure weight. In practice, one should probably include an allowance for the liner in calculating $\emptyset$. Therefore, the two numbers chosen as a basis for S probably represent, for these purposes, the extremes of the range of this factor that will apply in the next two or three-year period.

When they are examined for a particular value of density, the figures in Table II reveal the important effects of pressure. For a propellant with a density of 1.0 g/cm^3, for example, the range of the ΔI values when R is increased from 2 to 5 is as follows (in round figures):

$P(lb/in^2)$	100	200	500	1000
$S = 1.1 \times 10^{-4}$	2 - 3 sec	3 - 5	8 - 13	14 - 24
$S = 1.65 \times 10^{-4}$	2 - 4	4 - 8	11 - 19	21 - 35

The total spread in ΔI in this case is thus seen to be from 2 to 35 sec, that is, from a truly negligible figure to one that is so great as to put the propellant entirely out of serious consideration.

Pressure is by far the most important factor affecting ΔI. The effectiveness of a propellant with density 1.0 g/cm^3 would be seriously eroded if the pressure were much above 300 psi. Therefore, a propellant with this low density must be able to burn satisfactorily at pressures below 300 psi if it is to maintain essentially unimpaired the advantage it may have in specific impulse. In practice, this is probably not a severe requirement.

An analysis is made of the

VI. SYNOPSIS

Major goals in solid rocket propulsion development. ~~today are~~ These include high-specific impulse propellants and high propellant loading fraction. Progress in the second direction (b) will come through development of materials with higher strength/weight ratio, by the discovery of improved nozzle designs and throat materials, and -- possibly -- by development of cool-burning propellants with non-erosive, non-corrosive product gases and good low-pressure burning properties.

It is apparent that as Λ approaches more closely the limiting value of 1.0, that is, as the relative weight of structure compared to that of propellant approaches the vanishing point -- the effect of propellant density tends to disappear entirely. (This statement applies in the case of constant-weight constraint, which is the one of most concern where upper stages are concerned.) Actual calculations show (cf Table II) that even with presently attainable tankage-structure factors and combustion-chamber pressure levels the penalties for low propellant density are not large. Therefore, since the future trend will be to reduce the density effect still farther, there will come a time -- if it is not already here -- when concern for propellant density will be very slight indeed.

In fact, a point of steeply diminishing returns will also be reached in future efforts to increase the loading factor Λ; and this will mean more concentrated attention on the single goal of high specific

impulse to the exclusion of most other concerns. As new materials with better strength/weight are found, and as other improvements are made to reduce the structure factor $(1 - \Lambda)$, further improvements will obviously become increasingly difficult to make. Besides, a given percentage reduction in the structure fraction $(1 - \Lambda)$ has a smaller and smaller payoff in terms of equivalent specific impulse as Λ approaches closer and closer to 1.0.* Eventually, therefore, efforts in this direction will not be worth the candle.

We are therefore led to conclude that high specific impulse *per se* (pure and undiluted) is a goal worth pursuing in solid-propellant research. Special purposes may arise from time to time for propellants with high density, with very low burning rates, or with other unique characteristics; but, in general, there seems to be no valid way of -- or, indeed, any valid reason for -- lumping any of these properties with specific impulse to provide a criterion or figure of merit for solid propellants.

* This may be seen from the following expression (for constant-weight constraint):

$$\frac{\partial \ln I}{\partial \ln (1 - \Lambda)_{\Delta V, \zeta}} = \frac{R}{\ln R} \cdot (1 - \zeta)(1 - \Lambda)$$

EXPONENT n IN THE EXPRESSION $I_{SP} \rho^n$

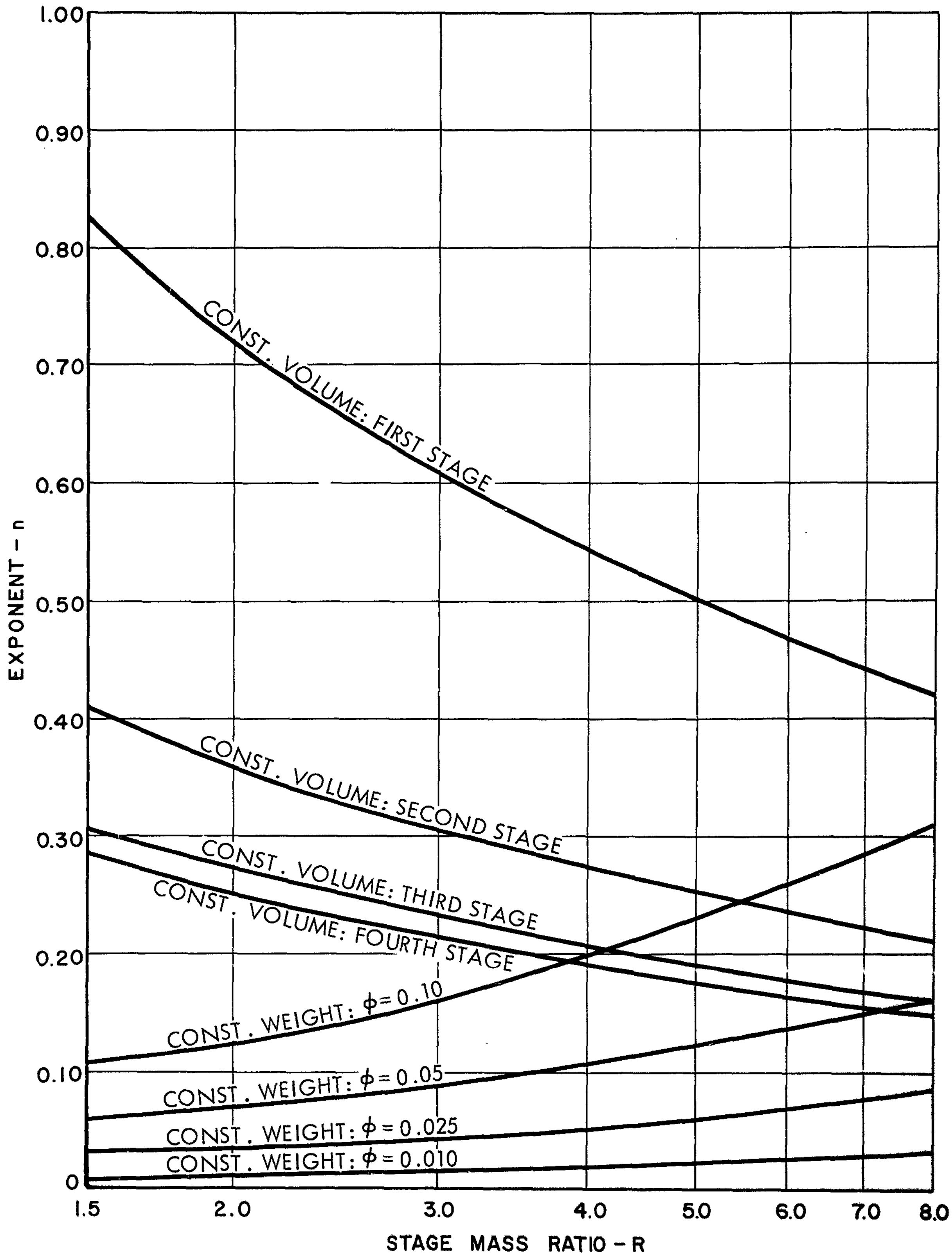

Figure 1